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Book Review

IMRE LAKATOS, *Proofs and Refutations*, Edited by JOHN WORRAL AND ELIE ZAHAR. Cambridge University Press, 1976, 174 pp.

This book is the most interesting work on the philosophy of mathematics to appear in many decades. It is not a technical contribution to the esoteric specialty called "foundations." Rather, it is a genuinely philosophical attack on the central question: What is the nature of mathematical knowledge? Every page is learned, witty, provocative, and controversial.

The introduction is a brilliant, slashing attack on dogmatic and formalist philosophies. The main text is a dialogue between an imaginary teacher and his students. The theme of the classroom dialogue, echoed and enriched in the historical footnotes, is the formula

$$V - E + F = 2$$

(Euler's formula for the number of vertices V, edges E, and faces F of a polyhedron). The footnotes are a historical counterpoint—a richly detailed and amazingly complex history of the Euler formula, starting not with Euler but with Descartes, and going on through Legendre, Cauchy, Crelle, Poinsot, L'Huilier, Gergonne, Mobius, Steinitz, and Poincaré.

The dialogue starts with a proof presented by the teacher. This is the traditional intuitive proof of Cauchy, in which the polyhedron is stretched on a plane, triangulated, and then simplified step by step. This is immediately followed by a whole menagerie of counterexamples, presented by the students. The debate is then, what did the proof prove? What do we "know" in mathematics?

As the controversy goes on, the theme is elaborated and developed: Mathematics is not a dogma sanctified by formal logic. It is a never-ending process, in which concepts are invented and their properties explored by a simultaneous search for proofs and refutations. The argument progresses from one surprise to another. The students argue with the teacher and with each other; each represents a distinct attitude toward mathematical truth. With frequent reversals of direction, new layers of ambiguity are revealed with each new clarification. The reader feels he is watching a magician who can always pull one more rabbit out of his sleeve.

Dialogues on mathematics have been written also by Renyi, by Galileo, and by Plato. Lakatos does not suffer by comparison.

On the explicit level, the goal of the discussion is a methodological analysis, a "logic of heuristic" to schematize the way mathematical knowledge advances

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by criticism and countercriticism, proof and disproof. Beneath this surface, there is implicit a philosophical viewpoint that incorporates mathematics into the framework of Karl Popper's critical philosophy, offering a new alternative to the dogmatic epistemology of all the "foundationist" schools (logicist, formalist, intuitionist). Instead of seeking to justify the assumed or demanded "absolute certainty" of true mathematical knowledge, Lakatos incorporates mathematics into a fallibilist philosophy of science. In mathematics as in natural science, knowledge progresses by criticism and countercriticism, and there is never a "final solution" that is "indubitable."

Implicit are new and fruitful answers to the basic questions: What is the content of mathematical statements? In what sense is mathematics knowledge *about* something? How do we attain it, and why do we believe in it?

"Proofs and Refutations" was first published in 1963, as a series of four articles in the *British Journal for the Philosophy of Science*. It was based on the first chapter of Lakatos' thesis, submitted at Cambridge in 1961 under the supervision of R. B. Braithwaite. As Lakatos wrote at that time, "The paper should be seen against the background of Polya's revival of mathematical heuristic, and of Popper's critical philosophy."

Some facts about Lakatos' life may help the reader to appreciate Lakatos' thinking. He was a survivor of the Nazi occupation of Hungary, a resistance fighter whose mother and grandmother perished at Auschwitz. In 1947, at the age of 25, he was virtually in charge of the reform of higher education in Hungary. Although an unwavering communist, he was arrested in 1950 and imprisoned until 1952. In 1954 Renyi used his influence to get him a job as a translator of mathematical books. One of the works he translated was Polya's "How to Solve It." At this time he became acquainted with the writings of Karl Popper as well. His communist certainties began to dissolve. After the Hungarian uprising of 1957, he fled to Vienna, and from there to England, where he met Polya and Popper. Polya suggested the history of the "Descartes–Euler conjecture" for his doctorate. In 1960 Lakatos joined Popper on the faculty of the London School of Economics. In February 1974, he died suddenly, at the age of 51. Except for a few brief fragments, he had published nothing more on mathematics. His work on the philosophy of science is well known, and he was editor of the *B.J.P.S.* at his death.

"Proofs and Refutations" marks a brilliant beginning. No doubt Lakatos hoped to return to this subject, to carry through the task laid out there. To make his views on the epistemology and ontology of mathematics explicit would mean, among many other things, spelling out the nature of informal mathematics and its relationship to formalized mathematics, to natural science, and to the physical world. Lakatos died leaving this task undone. It remains as a gigantic challenge for the present and future.

In preparing this book for posthumous publication, Worrall and Zahar have added valuable new material from Chapters 2 and 3 of Lakatos' thesis, dealing with Poincaré's proof of the Euler conjecture, with the history of the concept of uniform continuity, and with the implications of Lakatos' philosophical ideas for the teaching and exposition of mathematics. I suspect that much remains in Lakatos' unpublished manuscripts that would be of great interest and value to many readers.

It is a pity that Worral and Zahar felt it necessary to defend, in a sometimes gratuitous and embarrassing way, the dogmatic claims of formal logic against the skepticism of Lakatos. Everything is fallible, it seems, except formal logic. ("There is no serious sense in which such proofs are fallible," we read in a footnote by the editors on page 57.)

Worrall and Zahar would have done better to restrict their disagreements to the Preface. Their footnotes and starred additions are jarring and inappropriate.

Still, there is no question that in making "Proofs and Refutations" available at last in book form, they have rendered a great service to the mathematical community. It should be read by every student and practitioner of mathematics who has some curiosity to known what he is doing and why. For this, our hearty thanks to Professors Worrall and Zahar, to the Syndics of the Cambridge University Press, and to Lakatos' last research assistant, Gregory Currie.

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